

Coshocton.org-MX

Our 5 Number Worlds™: Measurement Scales Made Clear- Ogden Lindsley 1

1981

Fear not to Focus

In 1953 I focused my research and reading on human, free operant performance frequencies charted on standard charts (Lindsley, 1956). Following research and teaching leads expanded this focus so far that in 1970 I redesigned S. S. Stevens' scale system to simplify it for primary grade school children. Only short plain English words were used (Lindsley, 1991). This redesign has been shared with my students, but never before publicly presented.

Stevens' NOIR Ladder

In 1946 S.S. Stevens organized measurement scales - a system of how we use numbers (Stevens, 1946). His ladder, started with Nominal on the bottom, Ordinal on the second rung, Interval on the third, and Ratio on the top rung (the king of scales to Stevens). Reading up the ladder, students abbreviated Stevens' scale ladder NOIR, the French word for black, as a memory aid.

R	Ratio	Multiply interval
I	Interval	Add interval
O	Ordinal	Order number
N	Nominal	Name number

Widely covered in the statistical chapters of most psychology textbooks, NOIR has been accepted for organizing measurement scales for over 50 years - a half century!

NOIR Zero Problem

We faced two major problems teaching Stevens' NOIR scale ladder. The first was the Zero point.

On an interval scale Stevens said the zero point is arbitrary, as in temperature. Zero on the centigrade scale and zero on the fahrenheit scale are different and set by convention.

On a ratio scale Stevens said an absolute zero is always implied, even though it may never be reached.

The problem: If temperature is an interval scale how is it that there is an absolute zero temperature? According to Stevens an absolute zero would make temperature a Ratio scale. Student CONFUSION!

NOIR Ladder Problem

The second problem faced when teaching NOIR was the ladder problem.

If NOIR is a true hierarchy, as you go up the ladder each scale should include all the properties of the scales on rungs on the ladder below it. The statistics that may be used with nominal and ordinal scales should be all right to use with interval scales. This is correct.

But, the statistics correctly used with interval scales are NOT correct to use with Ratio scales on the rung above. The arithmetic mean, standard deviation, and product-moment correlation of interval scale data are not correct to use with ratio scale data. In fact the arithmetic average does not give a middle score on a ratio scaled set of numbers. You must use a multiply (geometric) average. Therefore, Nominal, Ordinal, and Interval are a ladder, but Interval and Ratio do not ladder. All the Interval procedures cannot be used correctly with a ratio scale. Student CONFUSION!

Lindsley's NO/AMP T

Solution to NOIR Ladder Problem

Our solution keeps the ladder of Name and Order numbers. There is no constant distance between numbers on these two scales.

Above Name and Order the distances (intervals) between the numbers on the scales have meaning - a constant add, multiply, or power.

Constant Distance (interval) Scales

20	1024	114 quadrillion
18	512	339 trillion
16	256	18 trillion
14	126	4 trillion
12	64	18 billion
10	32	4,294,967,296
8	16	65,536
6	8	256
4	4	16
2	2	4
0	1	2

Add	Multiply	Power
+2	x2	^2
Plus 2	Times 2	Times self

Our 5 Number Worlds™: Measurement Scales Made Clear- Ogden Lindsley 2

On our Add scale the distances are a constant: added going up or subtracted coming down the scale. A +2 add scale has numbers 0, 2, 4, 6, 8, 10, 12, 14 ...

On our Multiply scale the distances are a constant: multiplied going up or divided coming down. A x2 multiply scale has numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 ...

On our Power scale the distances are a constant: power going up or root coming down. A \wedge 2 power scale has numbers 2, 4, 16, 256, 65,536, 4,294,967,296 ... With first graders we call this a "times self" scale. Each constant distance is the number times itself.

We diagram the relationship between these 5 number worlds™ of Name, Order, Add, Multiply, and Power in a T form.

Add Multiply Power
Order
Name

We also diagram their initials as a T.

A M P
O
N

This diagram is written from its bottom up as NO/AMP and pronounced in English as "No amp."

The three rungs up this T ladder are a true hierarchy because statistics used on the Name rung can be used on the Order rung. The statistics from the Order rung can be used in all three scales on the AMP rung. Order world statistics can be used in the Add world, the Multiply world and in the Power world.

However, Add statistics are not correct to use in the Multiply world or Power world. Multiply world statistics (geometric mean) are not correct to use in the Add world where the add (arithmetic) mean must be used to give correct central values.

Solution to NOIR Zero Problem

The solution to the zero problem is found in the nature of the mathematics of our three constant distance (equal interval) number worlds.

Zero exists in our Add world. It is the operation for identity. You add zero to a number and it stays the same. You can start with zero and add 10 and you end up with 10.

Zero does not exist in our Multiply world. It is not a multiply number. When you multiply zero by a number, no matter how big it is, you can't get out of zero. You need a number bigger than zero to multiply from. That is why in solving factorials, if you run into a zero, the rule is to put the number 1 in zero's place. That is also why we cannot build behavior from zero frequency. We need at least one response to multiply by our reinforcers.

The number 1 is to our multiply world as zero is to our Add world - an identity operation. When you add zero or multiply by 1 the number stays the same.

Neither Zero nor 1 exist in our Power world. If we start with a number 1, no matter how big the power we raise it to, we can not get out of 1. So we need a number larger than 1 to raise to any power.

Separate 3 Things in Scale Thinking

When diagramming or thinking about scales and number worlds we must keep 3 things clearly separate. We must think of:

- 1) the scale Distances between the numbers,
- 2) the Numbers on the scale, and
- 3) how the thing being scaled Lives (grows, spreads, or is distributed).

Pure Scales

For clearest presentation, scale a thing with intervals, and scale numbers from the world in which it lives. If the thing grows by adding a constant, then it should be charted on an pure add scale with constant add intervals and add numbers. On a chart with time in an add scale across the bottom and the values in an add scale up the left, constant add growth forms a straight line.

Thermometers, rulers, and bathroom scales have pure add scales.

If the thing grows by multiplying by a constant (as do all living and behaving things) it should be charted on a pure multiply scale with both multiply distances and multiply numbers.

Mixed Scales

The common practice of putting add numbers on multiply distances confuses us. The logarithmic scale itself has add numbers (the characteristic) on the basic multiply distances (mantissa), as do the Decibels sound intensity, and Richter earthquake scales.

Statistical Formulas Between A, M, and P

Useful statistical rules occur between the Add, Multiply, and Power worlds. For example the general rule for finding an average or central value is:

- 1) put all the numbers together by the math of their world.
- 2) Bring it back to the size of one by borrowing the reducer from the next world to the right on the arm of the T.
- 3) Reduce using the number of numbers (N).

Example: To get an Add world mean (arithmetic mean):

- 1) Add all numbers together to get their sum.
- 2) Borrow the reducer divide from the multiply world to the right.
- 3) Divide the sum by N.

Example: To get a Multiply world mean (geometric mean):

- 1) Multiply all numbers together to get their product,
- 2) Borrow the reducer root from the power world to the right.
- 3) Root the product by N.

Real World Examples of Our 5 Number Worlds™

The scales we have discussed above are sub-classes of Our 5 Number Worlds. We have students sort cards with pictures of the following real world numbers into five boxes. Examples of the five different ways that we use numbers follow:

Name World Numbers

Sports jerseys
Telephone numbers
Automobile plates
Social Security numbers
Flight numbers

Order World Numbers

Sports seeds and rankings
IQ and Achievement Test scores
Course grades

Model and Serial numbers
House numbers
Interstate Highways

Add World Numbers

Sports game scores
Counters
Ruler
Thermometer
Bathroom scale
Clock
Speedometer and Altimeter
Altimeter
Checkbook balance
Street numbers (Chicago, NYC)
Correct and Error counts
L scale on slide rule
Distance and Time

Multiply World Numbers

(many are mixed scales)
Sports team and player improvement
Log, Decibel and Richter scale
Population and Disease
Percent
Interest and Dividends
Discount
Cost of living and Inflation
Performance and Learning
C and D scales on slide rule
All growth. All Decay

Power World Numbers

None so far

5 Number World Song (Tune: Three Blind Mice)

Five number worlds,
Five number worlds.
Name, Order, Add,
Multiply and Power,

We label our things in
our Name world.

We test ourselves in
our Order world.

We count everything in
our Add world.

Everything grows in
our multiply world.

And nothing yet lives in
our Power world.

Our Five number worlds.

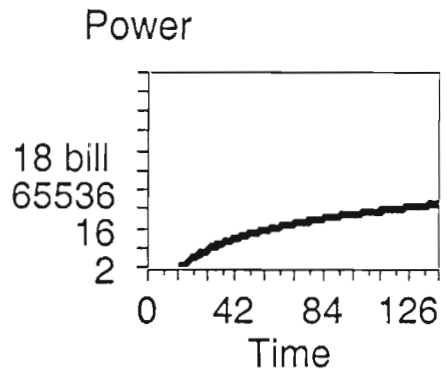
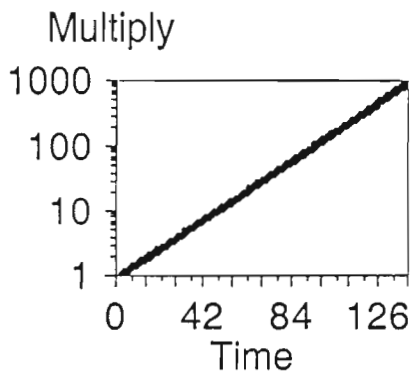
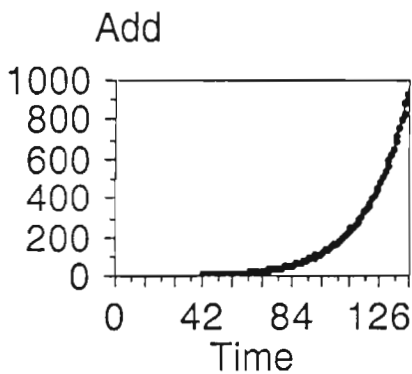
Mixed Scale Example - Richter +1 number x39 distance Earthquake Scale

Richter numbers tell the reader that a quake of 8 is about as much bigger than a 7, as a 4 (disturb objects) is of a 3 (felt indoors) - both +1 apart. Actually an 8 is the size of the 1906 San Francisco, and a 7 only collapses weak buildings. The energy in an 8 quake is 35 times greater than a 7 (Gere & Shah, 1984). Our school children learn earthquakes live in the multiply world, so the +1 numbers don't fool them.

Myth Log10	x39 Multiply distance Energy	+1 Add numbers Richter	Damage	Quakes
		10		
		9	Total destruction	Nuclear Bomb
x10	x31	8	Nearly total	1906 San Francisco
x10	x35	7	Collapse weak bldg	
x100 x1365	x39	6	Topple chimneys	Atomic Bomb
x10	x43	5	Crack walls	
x10	x48	4	Disturb objects	
		3	Felt indoors	
		2	Not felt	

x2 Time Series on Add, Multiply, and Power Charts

These three charts show x2 multiplying data set projecting easily in a straight line when charted on a multiply scale with multiply numbers. On an add scale it curves concave upwards like a cup. On a power scale it curves concave downwards like a dome. Moral: chart data the way they live.



REFERENCES

Gere, J. M. & Shah, H. C. (1984). *Terra non firma*. Stanford, CA: Stanford Alumni Association.
 Lindsley, O. R. (1956). Operant conditioning methods applied to research in chronic schizophrenia. *Psychiatric Research Reports*, 5, pp.118-139.
 Lindsley, O. R. (1991). From technical jargon to plain English for application. *Journal of Applied Behavior Analysis*, 24, 449-458.
 Stevens, S. S. (1946). On the theory of scales of measurement. *Science*, 103, 677-680.